The CENTRE for EDUCATION
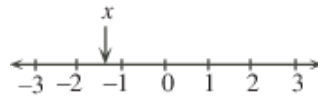## in MATHEMATICS and COMPUTING

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Topic Generator - Problem Set
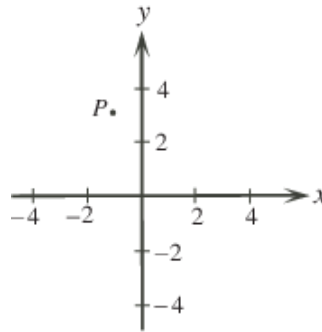**Problems**

1. The average (mean) of the numbers 6, 8, 9, 11, and 16 is
   (A) 8   (B) 9   (C) 10   (D) 11   (E) 7

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2. Which of the following is the best estimate for the value of x shown on the number line?

   (A) 1.3   (B) -1.3 (C) -2.7 (D) 0.7   (E) -0.7

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3. In the diagram, the coordinates of point P could be

   (A) $(1, 3)$ (B) $(1, -3)$ (C) $(-3, 1)$ (D) $(3, -1)$ (E) $(-1, 3)$

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4. The mean (average) of 5 consecutive integers is 9. What is the smallest of these 5 integers?
   (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

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5. Which of the following numbers is a multiple of 9?
   (A) 50   (B) 40   (C) 35   (D) 45   (E) 55

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6. The value of $2^4 - 2^3$ is

   (A) 0^1 (B) 2^1 (C) 2^2 (D) 2^3 (E) 1^1

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7. Which of the following numbers is greater than 0.7?

   (A) 0.07 (B) -0.41 (C) 0.8 (D) 0.35 (E) -0.9

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8. If $x = 3$, $y = 2x$, and $z = 3y$, then the average of x, y and z is

   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

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9. Alexis took a total of $243\,000$ steps during the 30 days in the month of April. What was her mean (average) number of steps per day in April?

   (A) 7900 (B) 8100 (C) 8000 (D) 7100 (E) 8200

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10. Elena earns $13.25 per hour working at a store. How much does Elena earn in 4 hours?

    (A) $54.00   (B) $56.25   (C) $52.25   (D) $51.00   (E) $53.00

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11. A student may pay $1.50 for a single bus ticket or $5.75 for a package of 5 tickets. If a student requires 40 tickets, how much does she save by buying all of the tickets in packages of 5 rather than buying 40 single tickets?

    (A) $54.25   (B) $34.00   (C) $14.00   (D) $8.25   (E) $4.25

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12. Sally picks four consecutive positive integers. She divides each integer by four, and then adds the remainders together. The sum of the remainders is

    (A) 6 (B) 1 (C) 2 (D) 3 (E) 4

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13. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?

    (A) 52 (B) 116 (C) 66 (D) 555 (E) 45

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14. Which of the following expressions is equal to 5?

    (A) $(2 \times 3)^2$ (B) $3 + 2^2$ (C) $2^3 - 1$

    (D) $3^2 - 2^2$ (E) $(3 + 2)^2$

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15. Which of the following numbers is closest to 1?

(A) $\frac{11}{10}$ (B) $\frac{111}{100}$ (C) 1.101 (D) $\frac{1111}{1000}$ (E) 1.011

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16. Which of the following is *not* equal to $\frac{15}{4}$?

(A) 3.75 (B) $\frac{14+1}{3+1}$ (C) $\frac{3}{4} + 3$ (D) $\frac{5}{4} \times \frac{3}{4}$ (E) $\frac{21}{4} - \frac{5}{4} - \frac{1}{4}$

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17. Integers greater than 1000 are created using the digits 2, 0, 1, 3 exactly once in each integer. What is the difference between the largest and the smallest integers that can be created in this way?

(A) 2187 (B) 2333 (C) 1980 (D) 3209 (E) 4233

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18. The operation $\otimes$ is defined by $a \otimes b = \dfrac{a}{b} + \dfrac{b}{a}$. What is the value of $4 \otimes 8$?

(A) $\dfrac{1}{2}$ (B) 1 (C) $\dfrac{5}{4}$ (D) 2 (E) $\dfrac{5}{2}$

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19. Abigail chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Bill chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Charlie chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. What is the probability that the product of their three integers is *not* a power of 2?

(A) $\frac{117}{125}$ (B) $\frac{2}{5}$ (C) $\frac{98}{125}$ (D) $\frac{3}{5}$ (E) $\frac{64}{125}$

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20. Juliana chooses three different numbers from the set $\{-6, -4, -2, 0, 1, 3, 5, 7\}$ and multiplies them together to obtain the integer n. What is the greatest possible value of n?

(A) 168 (B) 0 (C) 15 (D) 105 (E) 210

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21. If x and y are integers with $(y-1)^{x+y} = 4^3$, then the number of possible values for x is

(A) 8 (B) 3 (C) 4 (D) 5 (E) 6

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22. Four pieces of lumber are placed in parallel positions, as shown, perpendicular to line M:

- Piece W is 5 m long
- Piece X is 3 m long and its left end is 3 m from line M
- Piece Y is 5 m long and is 2 m from line M
- Piece Z is 4 m long and is 1.5 m from from line M

A single cut, perpendicular to the pieces of lumber, is made along the dotted line L. The total length of lumber on each side of L is the same. What is the length, in metres, of the part of piece W to the left of the cut?
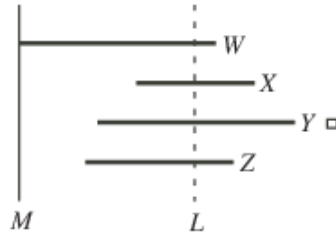
(A) 4.25   (B) 3.5   (C) 3.25   (D) 3.75   (E) 4.0

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23. A *Fano table* is a table with three columns where

- each entry is an integer taken from the list $1, 2, 3, \ldots, n$, and
- each row contains three different integers, and
- for each possible pair of distinct integers from the list $1, 2, 3, \ldots, n$, there is exactly one row that contains both of these integers.

The number of rows in the table will depend on the value of n. For example, the table shown is a Fano table with $n = 7$. (Notice that 2 and 6 appear in the same row only once, as does every other possible pair of the numbers $1, 2, 3, 4, 5, 6, 7$.) For how many values of n with $3 \leq n \leq 12$ can a Fano table be created?

1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

(A) 2   (B) 3   (C) 5   (D) 6   (E) 7

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24. Let n be the largest integer for which $14n$ has exactly 100 digits. Counting from right to left, what is the 68th digit of n?

(A) 1   (B) 2   (C) 4   (D) 5   (E) 8

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25. The number N is the product of all positive odd integers from 1 to 99 that do not end in the digit 5. That is,
$N = 1 \times 3 \times 7 \times 9 \times 11 \times 13 \times 17 \times 19 \times \cdots \times 91 \times 93 \times 97 \times 99$. The units digit of N is

(A) 1   (B) 3   (C) 5   (D) 7   (E) 9

26. The list of integers $4, 4, x, y, 13$ has been arranged from least to greatest. How many different possible ordered pairs (x, y) are there so that the average (mean) of these 5 integers is itself an integer?

(A) 7   (B) 8   (C) 9   (D) 10   (E) 11

27. There are n students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the n students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the n students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the n students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to $n^2 - n$ is

(A) 11   (B) 12   (C) 20   (D) 13   (E) 10

28. The number 385 is an example of a three-digit number for which one of the digits is the sum of the other two digits. How many numbers between 100 and 999 have this property?

(A) 144   (B) 126   (C) 108   (D) 234   (E) 64

29. It took Nasrin two hours and thirty minutes to canoe the 4.5 km into her camp. Paddling much faster, the return trip took her $\frac{1}{3}$ of the time. What was Nasrin's mean (average) speed as she paddled to camp and back?

(A) 1.25 km/h (B) 3.96 km/h (C) 1.8 km/h   (D) 1.95 km/h (E) 2.7 km/h

30. A *Pretti number* is a seven-digit positive integer with the following properties:

  - The integer formed by its leftmost three digits is a perfect square.

  - The integer formed by its rightmost four digits is a perfect cube.

  - Its ten thousands digit and ones (units) digit are equal.

  - Its thousands digit is not zero.

  How many Pretti numbers are there?